AP Physics
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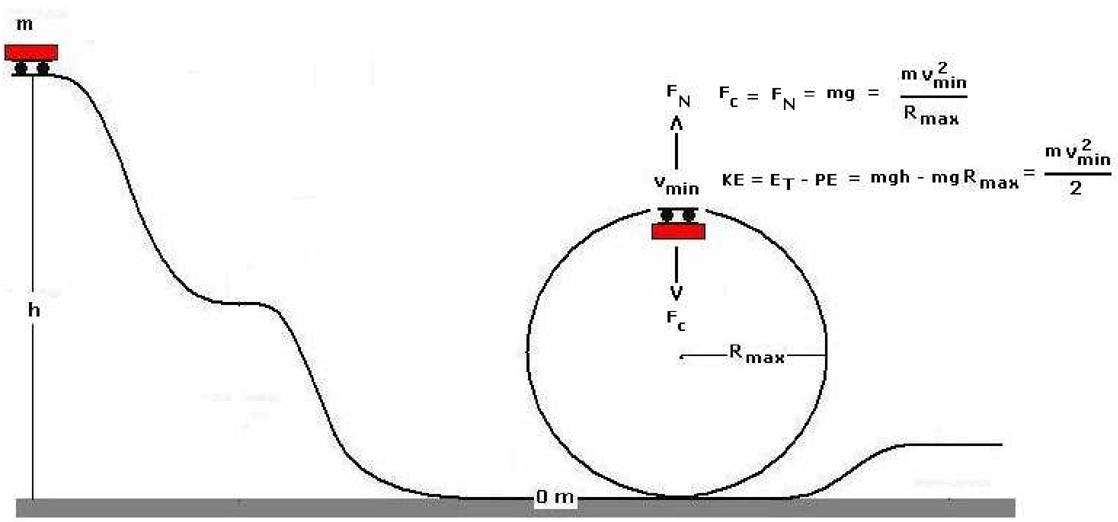
Derivation of the Equation for Calculating the Maximum Radius and Minimum Speed of Inverted Transit of a Roller Coaster Loop



A roller coaster of mass **m** at rest at the top of a frictionless roller coaster track of height **h** descends to base level and then enters an inverted circular loop. Derive the equations for calculating the maximum permissible radius [$\mathbf{R_{max}}$] of the inverted circular loop in order for the roller coaster to remain on the track and completely transit the loop.

At the top of the loop

$$\mathbf{F_c = -F_N = F_g}$$

where $\mathbf{F_c}$ is centripetal force, $\mathbf{F_N}$ is normal force and $\mathbf{F_g}$ is the weight of the roller coaster. Hence

$$\mathbf{F_c = mv_{min}^2/R_{max}}$$

is set equal to $\mathbf{F_N}$ which is equal in magnitude to $\mathbf{F_g}$ **:**

$$\mathbf{F_c = mv_{min}^2/R_{max} = F_N = mg}$$

where $\mathbf{v_{min}}$ in the minimum speed required for the roller coaster to remain on the track and completely transit the loop and
$\mathbf{g = 10.\ m/s^2}$.

Cancelling **m** from both sides yields

$$\mathbf{v_{min}^2/R_{max} = g}$$

Multipying both sides by $\mathbf{R_{max}}$ yields

$$\mathbf{v_{min}^2 = gR_{max}}$$

Taking the square root of both sides thence yields

$$\mathbf{v_{min} = [gR_{max}]^{1/2}} \qquad \textbf{[A]}$$

Considering the roller coaster in terms of its total [$\mathbf{E_T}$], gravitational potential [**PE**], and kinetic [**KE**] energies we have

$$\mathbf{E_T = KE + PE}$$

Solving for **KE**

$$\mathbf{KE = E_T - PE}$$

Substituting the expressions for each yields

$$\mathbf{mv_{min}^2/2 = mgh - mg[2R_{max}]}$$

Cancelling **m** from both sides yields

$$\mathbf{v_{min}^2/2 = gh - g[2R_{max}]}$$

and factoring out **g** from the right side of the equation yields

$$\mathbf{v_{min}^2/2 = g[h - 2R_{max}]}$$

Multiplying both sides by 2 yields

$$\mathbf{v_{min}^2 = 2g[h - 2R_{max}]}$$

Applying the distributive property yields

$$\mathbf{v_{min}^2 = 2gh - 4R_{max}}$$

Substituting equation **[A]** above

$$\mathbf{v_{min} = [gR_{max}]^{1/2}}$$

for $\mathbf{v_{min}}$ thence yields

$$\mathbf{([gR_{max}]^{1/2})^2 = gR_{max} = 2gh - 4R_{max}}$$

Adding $\mathbf{4R_{max}}$ to both sides of the equation yields

$$\mathbf{5gR_{max} = 2gh}$$

Dividing both sides of the equation by 5g thence yields

$$\mathbf{R_{max} = 2gh/5g}$$

Cancelling **g** from the right side of the equation hence yields

$$\mathbf{R_{max} = 2h/5} \qquad \textbf{[B]}$$

Therefore, the maximum radius [R_{max}] of an inverted circular loop is 0.4 of the starting height of any roller coaster [B].